

Dividing Polynomials Notes

Rules in Division of Polynomials

Rule 1: To divide monomials use law of exponent in division.

Rule 2: To divide polynomial by monomial, we use $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

Rule 3: The last rule is to divide a polynomial by another polynomial by another polynomial with at least two terms. This type of division is applied only when the degree of polynomial in the numerator is greater than or equal to the degree of polynomial in the denominator.

1. Arrange the term of both dividend and divisor in descending powers of the variable.
2. Divide the first term in the dividend by the first term of the divisor, giving the first term of the quotient.
3. Multiply each term of the divisor by the first term of the quotient and subtract the product from the dividend.
4. Use the remainder obtained in Step 3 as a new dividend, repeat Steps 2 and 3.
5. Continue the process until the remainder is reached whose degree should be less than the degree of the divisor.

The result of the division as follow:

A. For exact division (remainder 0)

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}(Q)$$

B. For remainder $\neq 0$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}(Q) + \frac{\text{remainder}}{\text{divisor}}$$

Sample Problem 1: Divide polynomials by monomials.

1. $24x^3 - 4x^2$ by $4x^2$

Solution:

$$\begin{aligned} &= \frac{24x^3 - 4x^2}{4x^2} \\ &= \frac{24x^3}{4x^2} - \frac{4x^2}{4x^2} \end{aligned}$$

$$= 4x - 1$$

2. $7x^5 - 2x^4 + 3x^2 + x$ by x

Solution:

$$= \frac{7x^5 - 2x^4 + 3x^2 + x}{x}$$

$$= \frac{7x^5}{x} - \frac{2x^4}{x} + \frac{3x^2}{x} + \frac{x}{x} = 7x^4 - 2x^3 + 3x + 1$$

$$= 7x^4 - 2x^3 + 3x + 1$$

Sample Problem 2: Divide polynomial by binomials.

3. $3x^3 + 7x^2 + 7x + 10$ by $x + 2$

Solution:

$$\begin{array}{r} 3x^2 + x + 5 \\ x+2 \overline{) 3x^3 + 7x^2 + 7x + 10} \\ \underline{3x^3 + 6x^2} \\ x^2 + 7x \\ \underline{x^2 + 2x} \\ 5x + 10 \\ \underline{5x + 10} \\ 0 \end{array}$$

$$\text{Answer: } 3x^2 + x + 5$$

4. $2x^3 + 6x^2 - 12x + 15$ by $2x + 2$

Solution:

$$\begin{array}{r} x^2 + 2x - 8 \\ 2x+2 \overline{) 2x^3 + 6x^2 - 12x + 15} \\ \underline{2x^3 + 2x^2} \\ 4x^2 - 12x \\ \underline{4x^2 + 4x} \\ -16x + 15 \\ \underline{-16x - 16} \\ 31 \end{array}$$

$$\text{Answer: } x^2 + 2x - 8 + \frac{31}{2x + 2}$$

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Synthetic Division

Another method of division which has a very short and simple procedure is called **synthetic division**. Unlike the usual division which involves the four fundamental operations. The division requires only *addition and multiplication* applied to coefficients. This method is applied when divisor is of the form $x \pm a$.

Steps to Follow in Synthetic Division

1. Arrange the term of the dividend in descending powers of the variable.
2. Write the numerical coefficients of each term of the dividend in row indicating the coefficients of powers. Replace the missing power by zero coefficients.
3. Replace the divisor $x - r$ by r : for divisor $x + r$, replace it by $-r$ (constant divisor).
4. Multiply the coefficient of the largest power of x , written on the third row, by the constant divisor. Place the product beneath the coefficient of the second largest power and add it to the coefficient. Multiply the sum of the constant divisor and place beneath the coefficient of the next largest power. Continue this procedure until there is a product added to the constant or last term.
5. The last number on the third row is called the remainder, and the rest of the numbers, starting from the left to right, are the coefficients of the terms in the quotient, which is one degree less than that of the dividend.

Sample Problem 3: Perform synthetic division to the following polynomials.

5. $(x^2 - x - 2)$ by $x + 1$

Solution:

$$\begin{array}{r} x^2 \quad x \quad c \end{array}$$

$$\begin{array}{r|rrr} 1 & 1 & -1 & -2 \end{array}$$

$$\begin{array}{r} \downarrow & -1 & 2 \end{array}$$

$$\begin{array}{r} 1 \quad -2 \quad 0 \rightarrow \text{remainder} \end{array}$$

Answer: $x - 2$

6. $4x^3 - 9x^2 - 3x + 5$ by $x + 2$

Solution:

$$\begin{array}{r} x^3 \quad x^2 \quad x \quad c \end{array}$$

$$\begin{array}{r|rrrr} 4 & -9 & -3 & 5 \end{array}$$

$$\begin{array}{r} \downarrow & -8 & 34 & -62 \end{array}$$

$$\begin{array}{r} 4 \quad -17 \quad 31 \quad -57 \rightarrow \text{remainder} \end{array}$$

Answer: $4x^2 - 17x + 31 - \frac{57}{x + 2}$